MODELING ON NONMATCHING GRIDS WITH APPROXIMATE DISTANCE FIELDS

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Enforcement of prescribed boundary conditions has emerged as a central issue in solving problems on grids that do not conform to the geometric domain. The challenges are particularly difficult for problems involving time-varying domains [1] and continuously heterogeneous materials [5]. We show that all types of boundary conditions may be satisfied precisely using approximate distance fields to the boundaries of the domains. Approximate distance fields behave as exact distance up to the specified order in the vicinity of the boundary of the geometric domain. Several methods can be used to construct approximate distance functions, including level set method, radial basis functions, adaptive sampling, and others. Our preferred construction method relies on R-functions that allows automatic construction of smooth fields from the standard geometric representations, such as boundary representations and Constructive Solid Geometry.

Once approximate distance fields are known, solutions to boundary value problems are represented via generalized Taylor series in terms of powers of the distance field. All boundary conditions are interpolated transfinitely via inverse weighting distance method [2], and the remainder term may be chosen to approximate differential or integral constraints – usually on a non-conforming grid of suitable shape functions, such as B-splines. Furthermore, the approximate distance fields are usually parameterized, and changes in the parameters are reflected not only in the distance fields but also in the resulting solutions to the boundary value problems. This makes our approach highly suitable for modeling and solving problems in time-varying geometric domains [1] and continuously heterogeneous materials [5].

The proposed approach is based on the Rvachev's Function Method (RFM) [3, 4] and has been fully implemented in the SAGE system that is available from http://sal-cnc.me.wisc.edu/Research/meshless. The typical solution procedure requires automatic differentiation, adaptive integration over non-meshed geometric object, solution of an algebraic problem and the visualization of the modeling results over the original geometry. We explain these steps and demonstrate the working computer system for meshfree analysis.

References

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